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Influence of a Kerr-Like Medium and the Detuning Parameter on the Statistical Aspects of the Intensity-Dependent-Coupling Hamiltonian

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We study the interaction between a one-mode electromagnetic field and a two-level atom in the presence of a Kerr-like medium when the atom is prepared initially in the momentum eigenstate. The wave function is calculated by using the Schrodinger equation for a coherent electromagnetic field and an atom in the excited state. The effects of the Kerr-like medium and the detuning parameters on the statistical aspects of the intensity-dependent-coupling Hamiltonian such as, atomic momentum increment, momentum diffusion, the radiation force, and the field entropy are calculated. We investigate the effect of the detuning, Kerr-like medium and photon number operator on the field entropy.

KEY WORDS: Kerr-like medium; intensity-dependent-coupling Hamiltonian.

1. INTRODUCTION

Recently, a large amount of work has been done on the atomic two-photon processes in a cavity both experimentally and theoretically. It is well known that, the two-photon processes in atomic systems are important in quantum optics due to the high degree of correlation between the emitted photons. This correlation leads to some interesting nonclassical effects in quantum optics. The Jaynes–Cummings model (JCM; Jaynes and Cummings, 1963) (for more references see Shore and Knight, 1993a) of a single two-level atom interacting with a single mode of the quantized radiation field is recognized as one of the most examined models in quantum optics. The JCM enables one to calculate all the quantum-mechanical properties of a system, and to predict many interesting effects such as vacuum field Rabi oscillations (Agarwal, 1984; Sanchez-Mondragon *et al.*, 1983; Shore and Knight, 1993b; Yoo and Eberly, 1985) quantum collapses and revivals of the atomic inversion (Dung *et al.*, 1990; Knight and Radmore, 1982; Milburn, 1984;

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Narozhny *et al.*, 1981), squeezed light (Kuklinski and Madajczyk, 1988; Meyster and Zubairy, 1982), Schrodinger cat states (Buzek, *et al.*, 1992; Yurke, *et al.*, 1990; Yurke and Stoler, 1986), and chaos (Fox and Eidson, 1986; Miloni *et al.*, 1983).

Over the years, the JCM have been extended and generalized in many directions. But in studies done on this topic, it is generally assumed that the atomic momentum and position could be classically considered so that the quantum effects on atomic external degrees of freedom, atomic internal degrees of freedom, and the traveling-wave field in a ring cavity, imposed by the initial field states and the initial statistics of momentum eigenstates, cannot be discussed. In this paper, an extension of the standard JCM has been made to include atomic external effects due to quantization of atomic motion; where the center-of-mass motion of an atom is cooled to extremely low temperature, so vibrational motion is quantized (Liu and Wang, 1996a). From the practical viewpoint, it is important to consider the cavity is filled with a Kerr-like medium. The Kerr nonlinearity corresponds to a Hamiltonian that is quadratic in the field number operator. Physically the model with Kerr-like medium may be realized as if the cavity contains two different species or Rydberg atoms of which one behaves like a two-level atom undergoing two-photon transition and the other behaves like an anharmonic oscillator in the single mode field of frequency ω_0 (Joshi and Puri, 1992; Obada *et al.*, 1999).

The JCM with intensity-dependent coupling has been discussed (Buck and Sukumar, 1981). This model is of interest because it gives rise to commensurable Rabi frequencies. The dynamical behavior of it is exactly periodic and can be compared with the standard JCM. Our aim in this paper is to study the effect of the Kerr-like medium and the detuning parameter on the collapses and revivals phenomena in the JCM with intensity-dependent coupling when the cavity is filled with a Kerr-like medium and obtain the time dependence of various statistical aspects such as atomic momentum increment, momentum diffusion, radiation force, and field entropy.

The paper is organized as follows. In Section 2, we present the intensity-dependent-coupling Hamiltonian to describe the interaction between two-level atom and one-mode field taking into account the Kerr-like medium. By using the Schrodinger equation we obtain the wave function when the atom is in the excited state. In Section 3, we calculate the expectation value of atomic momentum, momentum diffusion, the radiation force, and the field entropy when the field is in the coherent state. By a numerical computation, we examine the influence of the Kerr-like medium and the detuning parameter on the field entropy in Section 4. Finally, conclusions are presented.

2. DESCRIPTION OF THE MODEL

The study of the micromaser is an interesting field in quantum optics. Many quantum electrodynamics effects have been found in micromaser (Berman, 1994).

Recently, the ultracold atoms have been used instead of the thermal atoms in micromaser and the authros have proposed the concepts of mazer (Scully *et al.*, 1996). These studies treated the interaction between an incident atom in an excited state and a cavity field containing n photons, taking the quantum mechanical center-of-mass motion of the atom into account. Cold and ultracold atoms introduce new regimes in atomic physics often not considered in the past (Meyer *et al.*, 1997). It has been pointed out that in the Lamb–Dicke regime the dynamics of trapped ion can be described by a very simple Hamiltonian similar to that of the Jaynes-Cummings model (Shore and Knight, 1993a). Here we may refer to the interesting work given in Buzek *et al.*, (1997) where the authors considered the quantum motion of a cold, trapped two-level ion interacting with a quantized light field in a single-mode cavity. In this paper, we consider a similar situation but further we assume that the cavity is filled with a nonlinear medium such as a Kerr-like medium. We consider in an ideal cavity the interaction of a two-level atom with a single mode of inhomogeneous radiation field

$$E(x) = Re^{+i\vec{k}\cdot\vec{r}} + R^{+}e^{-i\vec{k}\cdot\vec{r}}. \tag{1}$$

The Hamiltonian of this model (with $\hbar = c = 1$) can be written in the rotating wave approximation (RWA) as

$$\hat{H} = \frac{\vec{P}^2}{2M} + \omega_0\sigma_0 + \omega_1\sigma_1 + \Omega\hat{a}^{+}\hat{a} + \xi\hat{a}^{+2}\hat{a}^2 + \lambda[R\sigma_{+}\, e^{+i\vec{k}\cdot\vec{r}} + R^{+}\sigma_{-}e^{-i\vec{k}\cdot\vec{r}}] \tag{2}$$

where $\vec{P}$ and $\vec{r}$ are the momentum and the position operators, $\vec{k}$ is the propagation vector, $\hat{a}(\hat{a}^{+})$ is annihilation (creation) of the one-mode field of the frequency Ω. The atomic energy levels are ω_0 and ω_1, λ is the coupling constant between the atom and the field, σ_0 and σ_1 are the ground and excited operators, $\sigma_{+}(\sigma_{-})$ are the raising (lowering) operators and ξ is the dispersive part of the Kerr-like medium. The operators R and R^{+} are given by the relations

$$R = af(\hat{n}) \quad \text{and} \quad R^{+} = f(\hat{n})a^{+} \tag{3}$$

where $f(\hat{n})$ being the real function of the photon number n $= \hat{a}^{+}\hat{a}$. Here we consider a special case of this Hamiltonian where $f(\hat{n}) = \sqrt{\hat{a}^{+}\hat{a}}$, which is the well-known bosonic operator (Buck and Sukumar, 1981; Buzek, 1989 a,b,c; Singh, 1982) describing the intensity dependent coupling the two-level JCM.

It is worthwhile remarking that investigating such models goes beyond an intrinsic theoretical interest because a new generation of high-Q electromagnetic cavities, covering a wide wavelength range, are today realizable (Yablonovitch *et al.*, 1992). Following Liu and Wang (1996b), the state vector of this model can be written as

$$|\Psi(t)\rangle = \sum_n q_n\big[A(t)\,e^{-i\gamma_1 t}|\vec{P}_{\text{o}}, n, e\rangle + B(t)e^{-i\gamma_2 t}|\vec{P}_{\text{o}}, n+1, g\rangle\big] \tag{4}$$

where

$$\gamma_1 = \frac{\vec{P}_o^2}{2M} + \omega_0 + \Omega n \quad \text{and} \quad \gamma_2 = \frac{\left(\vec{P}_o^2 + \vec{k}\right)^2}{2M} + \omega_1 + \Omega(n+1) \tag{5}$$

The expressions $|A(t)|^2$ and $|B(t)|^2$ represent the probabilities at time t. After some algebraic calculations we have

$$|\Psi(t)\rangle = \sum_n q_n \, e^{-it(\gamma_1 - \frac{\Delta}{2} + \frac{C_1+C_2}{2})} \left\{ \left[\cos(\mu t) - i \left(\frac{\Delta}{2} + \frac{C_1 - C_2}{2} \right) \frac{\sin(\mu t)}{\mu} \right] \right.$$

$$\left. \times |\vec{P}_o, n, e\rangle - iC\, e^{i\vec{k}\cdot\vec{r}} \, \frac{\sin(\mu t)}{\mu} |\vec{P}_o, n+1, g\rangle \right\}, \tag{6}$$

In the above equation

$$\mu^2 = \left(\frac{\Delta}{2} + \frac{C_1 - C_2}{2} \right)^2 + C^2, \qquad \Delta = \omega_0 - \omega_1 - \Omega - \frac{\vec{k}\cdot\vec{P}_0}{M} - \frac{\vec{k}^2}{2M},$$

$$C_1 = \xi n(n-1), \qquad C_2 = \xi n(n+1) \qquad C = \lambda(n+1) \tag{7}$$

The detuning parameter Δ depends on the recoil energy $\frac{\vec{k}^2}{2M}$ of the atom where M is the center of mass and the Doppler shift which is proportional to $\vec{k}\cdot\vec{P}_0$. Similarly, we can obtain the wave function of the atomic system when the atom is initially in the ground state. With the wave function $|\psi(t)\rangle$ calculated, any property related to this atomic system can be calculated.

3. THE STATISTICAL ASPECTS AND THE FIELD ENTROPY

In this section we shall calculate the expectation values of some operators such as atomic population $\langle\sigma_1\rangle$, atomic momentum increment $\langle\Delta\vec{P}\rangle = \langle\vec{P}\rangle - \vec{P}_0$, momentum diffusion $\langle(\Delta\vec{P})^2\rangle = \langle\vec{P}^2\rangle - \langle\vec{P}\rangle^2$, and radiation force $\langle\vec{F}\rangle = \frac{d}{dt}\langle\vec{P}\rangle$ when the atom is in the excited state $|e\rangle$ and the field in the coherent state $|q\rangle$ which could be expressed in terms of number states $|n\rangle$,

$$|q_n\rangle = \sum_n e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \tag{8}$$

By using the wave function Eq. (6), one obtains

$$\langle\Delta\vec{P}\rangle = \vec{k}\langle\sigma_1\rangle, \qquad \langle(\Delta\vec{P})^2\rangle = \vec{k}^2\langle\sigma_0\rangle\langle\sigma_1\rangle, \tag{9}$$

and

$$\langle\vec{F}\rangle = \vec{k} \sum_n |q_n|^2 C^2 \, \frac{\sin(2\mu t)}{\mu}, \tag{10}$$

where

$$\langle\sigma_1\rangle = \langle\psi(t)|\sigma_1|\psi(t)\rangle = \sum_n |q_n|^2 C^2 \frac{\sin^2(\mu t)}{\mu^2}, \tag{11}$$

and

$$\langle\sigma_0\rangle = \sum_n |q_n|^2 \left[1 - C^2 \frac{\sin^2(\mu t)}{\mu^2}\right], \tag{12}$$

In the following we shall use the field entropy as a measurement of the degree of entanglement between the atom and the field. Quantum mechanically the entropy is defined as (von Neumman, 1955)

$$S_f = -\mathrm{Tr}\{\rho_f \ \ln \rho_f\}, \tag{13}$$

where ρ is the density operator for a given quantum system which can be written as

$$\rho_f(t) = \mathrm{Tr}_{\mathrm{atom}}\{|\psi(t)\rangle\langle\psi(t)|\} = |A\rangle\langle A| + |B\rangle\langle B|, \tag{14}$$

where

$$|A\rangle = \sum_n q_n \, e^{-it(\gamma_1 - \frac{\Delta}{2} + \frac{C_1+C_2}{2})} \left[(\cos(\mu t) - i\left(\frac{\Delta}{2} + \frac{C_1 - C_2}{2}\right)\frac{\sin(\mu t)}{\mu}\right]|n\rangle, \tag{15}$$

$$|B\rangle = -\sum_n q_n \, e^{-it(\gamma_1 - \frac{\Delta}{2} + \frac{C_1+C_2}{2})} iC \, \frac{\sin(\mu t)}{\mu}|n+1\rangle. \tag{16}$$

We can go to a basis in which the density matrix of the field is diagonal and we can express the field entropy in terms of the eigenvalues $\lambda_f^{\pm}(t)$, of the reduced field density operator, as follows

$$S_f(t) = -\left[\lambda_f^+(t) \, \ln \lambda_f^+(t) + \lambda_f^-(t) \, \ln \lambda_f^-(t)\right]. \tag{17}$$

Now, we use the definition of the operator $p_f(t)$ to obtain the eigenvalues $\lambda_f^{\pm}$ as discussed in Phoenix and Knight (1990),

$$\begin{aligned}\lambda_f^{\pm} &= \langle A|A\rangle \pm \exp(\pm\theta)|\langle A|B\rangle| \\ &= \langle B|B\rangle \pm \exp(\mp\theta)|\langle A|B\rangle|\end{aligned} \tag{18}$$

where

$$\theta = \sinh^{-1}\left[\frac{1}{2\langle A|B\rangle}(\langle A|A\rangle - \langle B|B\rangle)\right], \tag{19}$$

where $\langle A|A\rangle$, $\langle B|B\rangle$, and $\langle A|B\rangle$ can be calculated from Eqs. (15) and (16).

In what follows we investigate numerically the effect of Kerr-like medium, detuning parameter and photon number operator on the dynamical behavior of the field entropy of the system.

4. RESULTS OF CALCULATIONS

In this section, we investigate the effect of the detuning and Kerr-like medium on the considered system with $\bar{n} = 10$ in the presence and absence of the photon number operator ($f(\hat{n}) = \sqrt{\hat{a}^{+}\hat{a}},\ f(\hat{n}) = 1$) for the field entropy. In Fig. 1, we plot the entropy of the field against the scaled time in the absence of the medium ($\xi = 0$) and photon number ($f(\hat{n}) = 1$) with $\Delta = 0, 5, 10$ corresponding to the curves (a), (b), and (c), respectively. We notice that the maximum value of the entropy is approximately 0.7, and this value is affected by the detuning. Also, the entropy shows fluctuations after a certain period of time. By increasing the detuning as shown in Fig. 1 (b,c) the value of maximum is decreased in addition fluctuations start after a longer period time by comparison with the state of absence the detuning as in Fig. 1(a).

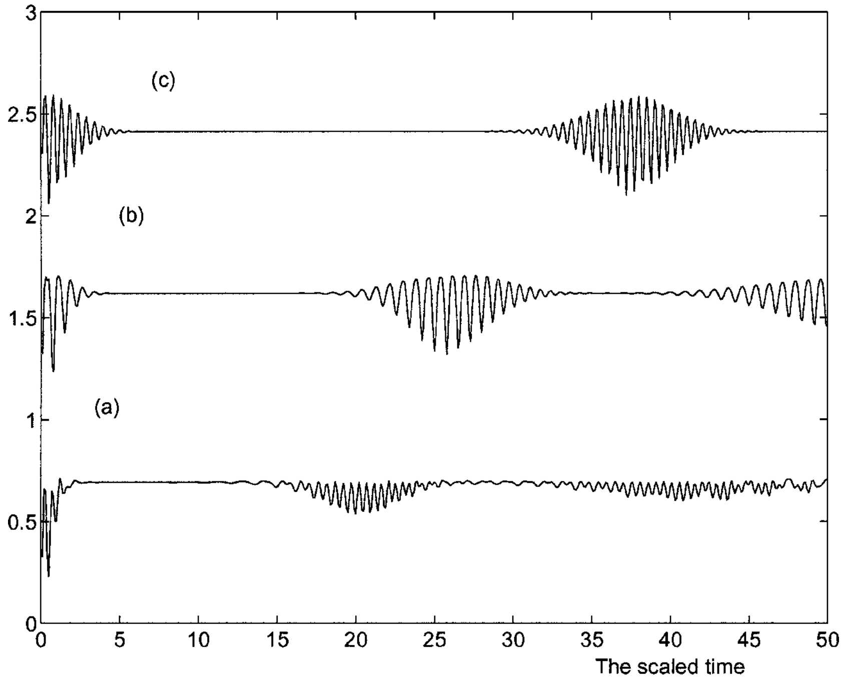

Fig. 1. The field entropy against the scaled time with $\bar{h} = 10$, $\xi = 0$, and $f(\hat{n}) = 1$ for different values of the detuning parameter: (a) $\Delta = 0$, (b) $\Delta = 5$, (c) $\Delta = 10$.

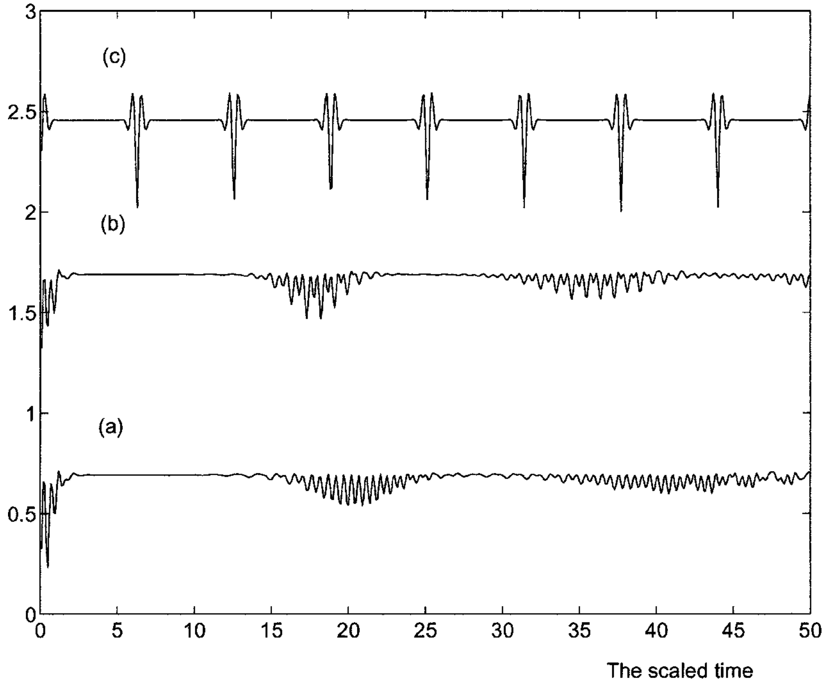

Fig. 2. The same as in Fig. 1 but $\Delta = 0$ for different values of the Kerr-like medium: (a) $\xi = 0.01$, (b) $\xi = 0.1$, (c) $\xi = 0.5$.

In Fig. 2, we study the effect of the Kerr-like medium (with $\xi = 0.01$, 0.1, 0.5, and $f(\hat{n}) = 1$) on the entropy. We notice that the entropy is affected by the Kerr-like medium. When $\xi = 0.01$ the effect is weak (see Fig. 1(a) and Fig. 2(a)). However, when $\xi = 0.1$ the fluctuations of the entropy decrease and its period time also decrease; but when $\xi = 0.5$ the fluctuations occur for a short time but more periodically as shown in Fig. 2(c).

To study the effect of the photon number operator on the entropy, we plot Fig. 3 with the same as in Fig. 1 and Fig. 2 but $f(\hat{n}) = \sqrt{\hat{a}^{+}\hat{a}}$. We observe that the entropy is affected by the function $f(\hat{n})$ as follows: the fluctuations occur for a small time and more oscillations at the same period of time have been observed. The field entropy reaches to the pure state when $\Delta = 0$ at $\lambda t \simeq 37.5$. The results here are in agreement with the results obtained in Abdel-Aty (2002), where the author has studied the general formalism for a two-level atom multiphoton process, taking into account arbitrary forms of nonlinearities of both the field and the intensity-dependent-atom-field coupling. The work here extends previous studies in this context. For the resonant case $\Delta = 0$, the quantum field entropy reaches its minimum value at $\lambda t \cong 37.5$ (Fig. 4).

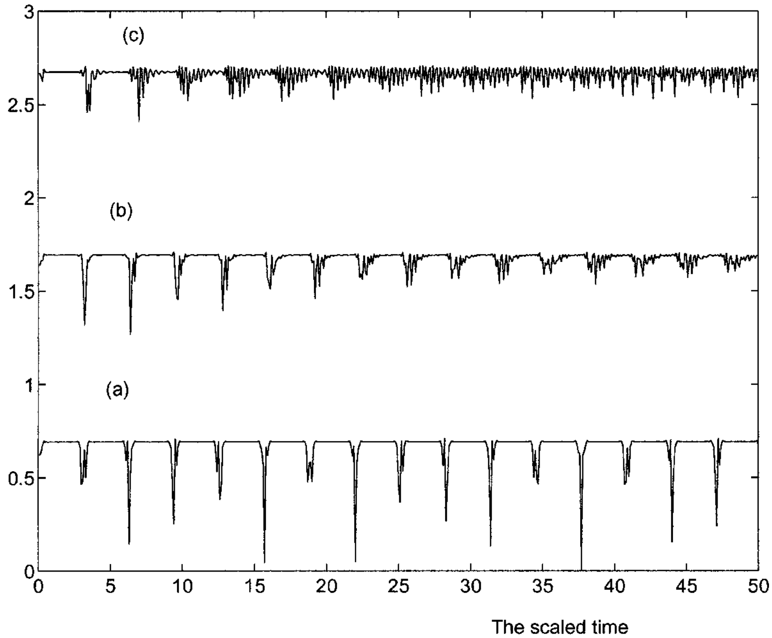

Fig. 3. The same as in Fig. 1 but $f(\hat{n}) = \sqrt{\hat{a}^{+}\hat{a}}$.

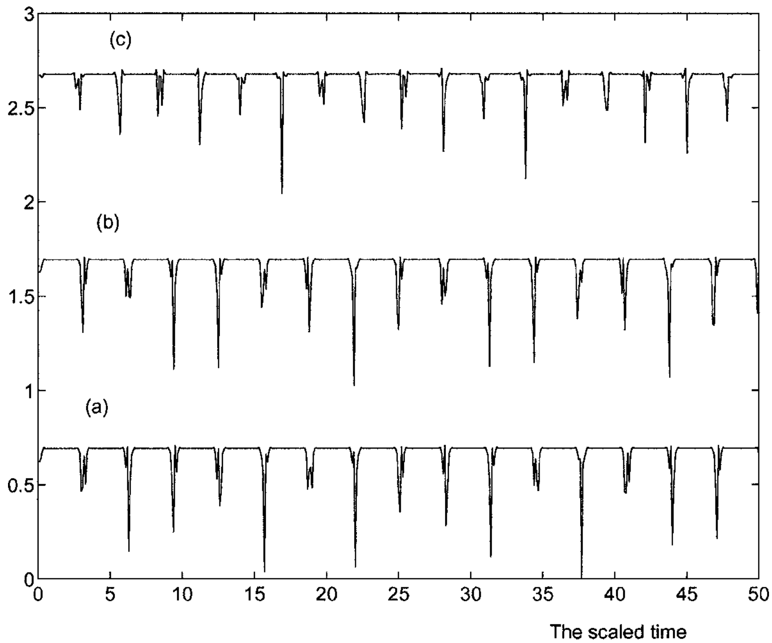

Fig. 4. The same as in Fig. 2 but $f(\hat{n}) = \sqrt{\hat{a}^{+}\hat{a}}$.

In conclusion, we studied the interaction between one-mode cavity field and two-level atom in a pure momentum eigenstate in the presence of a Kerr-like medium. We obtained the wave function of this atomic system and calculated some aspects related to this system such as momentum increment, momentum diffusion, radiation force, and the entropy. The effects of the detuning and the Kerr-like medium on the field entropy are investigated. It is found that the field entropy has the quantum collapses–revivals behavior. It is noticed that the intensity-dependent medium modifies the effective detuning to effectuate the modification of the collapse–revival phenomena in the atomic motion. But in the intensity-dependent medium, the collapse–revival phenomena are distorted due to the huge change of both the effective detuning and effective photon statistics.

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